Hilfs-Integral um Fourierkoeffizieten von $u_0(x)$ zu bestimmen.

$$u(x_0) = \begin{cases} 2x; & 0 \leq x \leq 1/2 \\ 2 - 2x; & 1/2 \leq x \leq 1 \end{cases}$$

$$\int_0^1 u(x_0) sin(k\pi x)$$

1. $\displaystyle\int_0^{1/2} x\, sin(k\pi x)\, dx =$

$$-\left[\frac{x}{k\pi} cos(k\pi x)\right]_0^{1/2} - \frac{1}{k\pi}\int_0^{1/2} x\,(-cos(k\pi x))\, dx =$$

$$-\frac{1}{2k\pi} cos(\tfrac{k\pi}{2}) + \left[\frac{1}{k^2\pi^2} sin(k\pi x)\right]_0^{1/2} =$$

$$-\frac{1}{2k\pi} cos(\tfrac{k\pi}{2}) + \frac{1}{k^2\pi^2} sin(\tfrac{k\pi}{2})$$

2. $\displaystyle\int_{1/2}^1 sin(k\pi x)\, dx = -\left[\frac{1}{k\pi} cos(k\pi x)\right]_{1/2}^1 =$

$$-\frac{1}{k\pi} cos(k\pi) + \frac{1}{k\pi} cos(\tfrac{k\pi}{2})$$

3. $\displaystyle -\int_{1/2}^1 x\, sin(k\pi x)\, dx = -\left[\frac{x}{k\pi} cos(k\pi x)\right]_{1/2}^1 + \left[\frac{1}{k^2\pi^2} sin(k\pi x)\right]_{1/2}^1 =$

$$-\frac{1}{k\pi} cos(k\pi) - \frac{1}{2k\pi} cos(\tfrac{k\pi}{2}) - \frac{1}{k^2\pi^2} sin(k\pi) + \frac{1}{k^2\pi^2} sin(\tfrac{k\pi}{2})$$

1. + 2. + 3. =

$$\frac{2}{k^2\pi^2} sin(\tfrac{k\pi}{2})$$

$$\Rightarrow \int_0^1 u(x_0) sin(k\pi x) = \frac{4}{k^2\pi^2} sin(\tfrac{k\pi}{2})$$